Algorithms for Massive, Expensive, and Otherwise Inconvenient Graphs

DAVID TENCH
UNIVERSITY OF MASSACHUSETTS AMHERST
Ask questions if you’re confused!
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Much of this presentation requires basic CS knowledge only.
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I’ll warn you before the tricky parts.
Algorithms for Massive, Expensive, and Otherwise Inconvenient Graphs

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Convenient Inputs
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• What is the 4\textsuperscript{th} element of this list?
Convenient Inputs

• What is the 4th element of this list?
• Is there an edge between some pair of nodes in this graph?
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An algorithm can access any part of its input at any time at unit cost.
Convenient Inputs

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• Is there an edge between some pair of nodes in this graph?

An algorithm can access any part of its input at any time at unit cost.

This is the random access property.
When Random Access Fails
When Random Access Fails

When inputs are too large to fit in memory
When Random Access Fails

When inputs are too large
to fit in memory
When Random Access Fails

When inputs are too large to fit in memory

When parts of inputs are costly to discover
When Random Access Fails

When inputs are too large to fit in memory

When parts of inputs are costly to discover
Overview of this talk

Streaming graph algorithms:
Coping with graphs too large to fit in memory
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Streaming graph algorithms: Coping with graphs too large to fit in memory

Collaborations with practitioners: Graph algorithms subject to expensive edge queries
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Streaming graph algorithms: Coping with graphs too large to fit in memory

Collaborations with practitioners: Graph algorithms subject to expensive edge queries

(Also future work)
Streaming
COMPUTING WITH INCREDIBLY LARGE INPUTS
When Graphs Are Too Large

Can’t store graph in memory
When Graphs Are Too Large

Can’t store graph in memory

Receive stream of edges
When Graphs Are Too Large

Can’t store graph in memory

Receive stream of edges

• Vertex Connectivity (PODS 2015)
• Densest Subgraph (MFCS 2015)
• Unique Cover (current work)
Warm-up: Missing Number

I read you an unordered list of all the integers from 1 to 5 million – except I leave one of them out. After, I ask you which is missing.

You only have a single piece of paper and a pencil. How do you find the missing number?
Warm-up: Missing Number

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Answer: keep a running sum of all the numbers, then subtract from $1 + 2 + ... + 5$ million.
Graph Streaming
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Facebook has almost 2 billion users
Graph Streaming

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Nodes = users, edges = friend relationships
Graph Streaming

Facebook has almost 2 billion users
Nodes = users, edges = friend relationships
This graph could have almost \((2 \text{ billion})^2 = 4 \text{ quintillion edges}\)
Graph Streaming

Facebook has almost 2 billion users
Nodes = users, edges = friend relationships
This graph could have almost
$(2 \text{ billion})^2 = 4 \text{ quintillion edges}$
For modern computers, storing 2 billion objects is maybe reasonable but 4 quintillion is not
Graph Streaming

It’s possible to store some of the edges, but not all
Graph Streaming

It’s possible to store some of the edges, but not all.

We have roughly $n$ space, where $n = \# \text{ of nodes}$
Graph Streaming

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We have roughly $n$ space, where $n = \# \text{ of nodes}$.

Graph edges are given as a stream.
Defining a graph via a stream
Defining a graph via a stream

Add edge (1,4)
Defining a graph via a stream
Defining a graph via a stream

Add edge (3,4)
Defining a graph via a stream
Defining a graph via a stream

Add edge (1,3)
Defining a graph via a stream
Defining a graph via a stream

Add edge (2,5)
Defining a graph via a stream
Defining a graph via a stream

Stream:
- Add edge (1,4)
- Add edge (3,4)
- Add edge (1,3)
- Add edge (2,5)

Resulting Graph:
Defining a graph via a stream

Stream:
Add edge (1,4)
Add edge (3,4)
Add edge (1,3)
Add edge (2,5)

Resulting Graph:

Edge deletions are also possible, but we’re ignoring them today.
Problem: Vertex Connectivity

What is the minimum number of nodes we can remove to disconnect a graph $G$?
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Problem: Vertex Connectivity

What is the minimum number of nodes we can remove to disconnect a graph \( G \)?
Problem: Vertex Connectivity

What is the minimum number of nodes we can remove to disconnect a graph G?
Problem: Vertex Connectivity

What is the minimum number of nodes we can remove to disconnect a graph G?

How many computers in a network can fail before the remaining network is disconnected?
Problem: Vertex Connectivity

What is the minimum number of nodes we can remove to disconnect a graph $G$?

Easily solvable using max flow algorithm, but this requires random access to the graph.
Problem: Vertex Connectivity

We show how to create a certificate graph $H$ that matches $G$’s vertex connectivity up to constant $k$, but has only roughly $kn$ edges.
Problem: Vertex Connectivity

We show how to create a certificate graph $H$ that matches $G$’s vertex connectivity up to constant $k$, but has only $kn$ edges.

We also show how to $(1+\epsilon)$-approximate vertex connectivity while only storing $\epsilon^{-1}kn$ edges.
Problem: Vertex Connectivity

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Finally, we show how to construct hypergraph sparsifiers in roughly linear space.
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Warm-up: Is the Graph Connected?

Original graph

Summary
Warm-up: Is the Graph Connected?

As each edge arrives in the stream, we keep it only if its endpoints were not already connected.
Warm-up: Is the Graph Connected?

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This is a **spanning forest** and has at most n-1 edges.
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This is a **spanning forest** and has at most \( n-1 \) edges.

If the spanning forest is connected, we know the original graph was as well.
Back to Vertex Connectivity
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First we delete each node in $G$ with probability $1-1/k$. 
Back to Vertex Connectivity

First we delete each node in G with probability $1 - 1/k$. 
Back to Vertex Connectivity

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During the stream we maintain a spanning forest $T$ on the remaining nodes.
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During the stream we maintain a spanning forest $T$ on the remaining nodes.

Repeat roughly $k^2$ times in parallel* to make $T_1$, $T_2$, …
Back to Vertex Connectivity

After the stream, merge all $T_i$ to form $H$, our vertex connectivity certificate.
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Each spanning forest has $n/k$ edges in expectation and there are $k^2$ of them so $H$ has $kn$ edges in expectation.
Back to Vertex Connectivity

After the stream, merge all $T_i$ to form $H$, our vertex connectivity certificate.

Theorem: For some arbitrary set $S$ of at most $k$ nodes, $G \setminus S$ is disconnected iff $H \setminus S$ is disconnected.
Proof Sketch of Theorem

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To prove: When $G\setminus S$ is disconnected, $H\setminus S$ must be disconnected

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To prove: When \( G \setminus S \) is disconnected, \( H \setminus S \) must be disconnected.

\( H \) is a subgraph of \( G \), so if \( G \setminus S \) lacks a path between nodes \( u \) and \( v \), so does \( H \setminus S \).
Proof Sketch of Theorem

To prove: When $G \setminus S$ is disconnected, $H \setminus S$ must be disconnected

$H$ is a subgraph of $G$, so if $G \setminus S$ lacks a path between nodes $u$ and $v$, so does $H \setminus S$.

With high probability, $H$ has all of $G$'s nodes. //
Proof Sketch of Theorem

To prove: When $G \setminus S$ is connected, $H \setminus S$ must be connected
Proof Sketch of Theorem

To prove: When $G\setminus S$ is connected, $H\setminus S$ must be connected.
To prove: If some edge $(u,v)$ exists in $G\setminus S$, then there is a path between $u$ and $v$ in $H\setminus S$. 
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G\setminus S\), \(u\) and \(v\) are connected in \(H\setminus S\).
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G\backslash S\), \(u\) and \(v\) are connected in \(H\backslash S\).

If there's some \(T_i\) that contains both \(u\) and \(v\)...
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G \setminus S\), \(u\) and \(v\) are connected in \(H \setminus S\).

If there’s some \(T_i\) that contains both \(u\) and \(v\)…

Either edge \((u,v)\) is in \(T_i\) or
Proof Sketch of Theorem

To prove: If edge (u,v) exists in G\S, u and v are connected in H\S.

If there’s some \( T_i \) that contains both u and v…

Either edge (u,v) is in \( T_i \) or

Some other path between u and v is in \( T_i \)
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G\setminus S\), \(u\) and \(v\) are connected in \(H\setminus S\).

If there’s some \(T_i\) that contains both \(u\) and \(v\)...

The only potential problem is that this alternate path may contain a node that’s in \(S\) (red node).
Proof Sketch of Theorem

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If there’s some \(T_i\) that contains both \(u\) and \(v\)...

The only potential problem is that this alternate path may contain a node that’s in \(S\) (red node).

So we need \(T_i\) to not contain any nodes in \(S\).
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G \setminus S\), \(u\) and \(v\) are connected in \(H \setminus S\).

We need \(T_i\) to contain \(u\) and \(v\), and no red \(S\) nodes.

\[
P(u \text{ and } v \text{ connected in } T_i \setminus S) = \frac{1}{k^2} \left( 1 - \frac{1}{k} \right)^k
\]
Proof Sketch of Theorem

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$$P(u \text{ and } v \text{ disconnected in } T_i \setminus S) = 1 - \frac{1}{k^2} \left(1 - \frac{1}{k}\right)^k$$

$$P(u \text{ and } v \text{ disconnected in } H \setminus S) = \left(1 - \frac{1}{k^2} \left(1 - \frac{1}{k}\right)^k\right)^{O(k^2 \log(n))} \leq \frac{1}{n^4}$$
Proof Sketch of Theorem

To prove: If edge \((u,v)\) exists in \(G\setminus S\), \(u\) and \(v\) are connected in \(H\setminus S\).

We need \(T_i\) to contain \(u\) and \(v\), and no red \(S\) nodes.

The probability that no \(T_i\) meets this requirement is at most \(1/n^4\).

So \(u\) and \(v\) are connected in \(H\setminus S\) with high probability. //
We did it!

We showed how to create a *certificate* graph $H$ that matches $G$’s vertex connectivity up to constant $k$, but has only roughly $kn$ edges.
We did it!

We showed how to create a certificate graph $H$ that matches $G$’s vertex connectivity up to constant $k$, but has only roughly $kn$ edges.

You can process a massive stream of the edges in $G$ to create $H$, which is much smaller. Then you can run a traditional vertex connectivity algorithm on $H$ to get your answer.
Query-Based Algorithms

WHEN DISCOVERING GRAPH EDGES IS COSTLY
When Graph Edges Are Costly

Can check existence of any edge at any time, but pay a significant cost
Want to minimize # of queries
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Mesh memory manager (PLDI 2019)
When Graph Edges Are Costly

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Mesh memory manager (PLDI 2019)
PathCache network path predictor (to be submitted SIGCOMM 2020)
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Memory Fragmentation
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We want to reorganize or “compact” the allocated regions of memory to be contiguous.
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We want to reorganize or “compact” the allocated regions of memory to be contiguous.
Modern operating systems maintain a mapping between virtual and physical memory.
If we relocate objects in physical memory, we have to update their virtual addresses as well.
Virtual Memory – A Quick Primer

But in C and C++, we can’t alter virtual addresses safely.
How can we relocate objects without changing their virtual addresses?
Virtual Memory – A Quick Primer

We can remap two virtual pages onto the same physical page in memory*, and discard one of the physical pages.
Virtual Memory – A Quick Primer

We can remap two virtual pages onto the same physical page in memory*, and discard one of the physical pages.

* provided there are no collisions between objects on the two pages.
- Memory organized into pages
- Each page holds same # of objects (8)
- Objects are placed on page uniformly at random
We can represent each page as a bitstring, where 0 indicates a free slot and 1 indicates an occupied slot.

We can *mesh* two pages together if they don’t have 1s in the same position.
We can represent each page as a bitstring, where 0 indicates a free slot and 1 indicates an occupied slot.

We can *mesh* two pages together if they don’t have 1s in the same position.
THEY MESH!
If both bitstrings have a 1 in some position, we can’t mesh the strings together.
NO MESH
Now we can forget about the details of memory, and think about our problem in terms of finding meshable pairs of bitstrings.

We want to mesh as many pairs of strings as possible.
We can think of this as a graph problem!
We can think of this as a graph problem!
Mesh as many pairs as possible.
Maximum Matching
Maximum Matching

Well-known polynomial time algorithm
Maximum Matching

- Well-known polynomial time algorithm
- Requires random access to graph
Maximum Matching

Well-known polynomial time algorithm

Requires random access to graph

Do we have random access?
No.
Not Enough Time!

We mesh during program execution.
Not Enough Time!

We mesh **during program execution**.

We must “pause” the program to mesh.
Not Enough Time!

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Only safe to “pause” for a very short time.
Not Enough Time!

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Graph doesn’t exist yet! We have to do computation to test whether any edge exists, costing valuable time.
We mesh **during program execution**.

We must “pause” the program to mesh.

Only safe to “pause” for a very short time.

Graph doesn’t exist yet! We have to do computation to test whether any edge exists, costing valuable time.

Checking an edge is a costly query.
Random Graphs

Recall that the 1s in the bitstrings are distributed randomly.

0000100000011000
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Say we have a length b bitstring that’s 10% full.
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We can write the probability that it meshes with another 10% full string as

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\[ q = \binom{b - b/10}{b/10} / \binom{b}{b/10} \]

Say we have a length \( b \) bitstring that’s 10% full.
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In general, if we know how many 1s are in each string, we can get the probability of any two strings meshing.

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Recall that the 1s in the bitstrings are distributed randomly.

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We can expect to find a mesh for a bitstring using $1/q$ queries.
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But the situation isn’t so simple.
Random Graphs

There is a rich body of work on algorithms for random graphs (including matching).

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That’s not the case for meshing graphs!

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Edge Dependence Example

Say we know all of our bitstrings are half full.
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Then if two bitstrings mesh, they must be each other’s complement.
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0110 — 1001

???? (50% full)
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So in this case triangles are impossible. If two of the edges exist, the third must not exist.
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This is a novel and interesting mathematical structure!
Mesh

We built Mesh, a memory manager powered by a query-limited matching algorithm that can perform memory compaction in C and C++. 
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It is freely available on Github.
Finding the Shape of the Internet

I want to visit www.colgate.edu. What path through the internet will my HTTP request take?
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I want to visit www.colgate.edu. What path through the internet will my HTTP request take?

How can I predict paths from any starting location to colgate.edu?
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How can I predict paths from any starting location to colgate.edu?

Measuring a path is a costly query called a *traceroute*. We want to discover as much of the Internet as possible using minimal traceroutes.
PathCache

A system for efficiently using limited VP measurements to predict paths towards Internet destinations.

Current version discovers 4 times more connections than pre-existing measurement strategies with comparable traceroute budget.

Predicts correct or nearly-correct paths 75% of the time.
Future Work

GRAPHS THAT ARE EVEN MORE INCONVENIENT
Time-Dependent Graph Streams

In the typical streaming model, graph is the same regardless of the order edges appear in the stream.

What if the order mattered?

Ex: disease spreading
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Ex: disease spreading

Temporal graphs are just beginning to be investigated. No streaming work.
Imagine we receive a massive stream of handshakes between many people. Later, we learn one of those people was sick.

Can we determine who is infected without storing the entire stream?
Time-Dependent Graph Streams

This problem is about connectivity or reachability on temporal graphs.
Time-Dependent Graph Streams

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Other potential problems:
Time-Dependent Graph Streams

This problem is about connectivity or reachability on temporal graphs.

Other potential problems:

Determine how long it takes for a disease (or information, or goods) to reach every part of a network.
Time-Dependent Graph Streams

This problem is about connectivity or reachability on temporal graphs.

Other potential problems:

Determine how long it takes for a disease (or information, or goods) to reach every part of a network.

Estimate how many different Patient Zeros could infect a particular person.
Reconstructing Graphs

We want to reconstruct a graph $G$. We can make a query which returns a random, unlabeled induced subgraph of $G$. How many queries are needed?
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Reconstruction from other queries?
Thanks for listening!